California Subject Examinations for Teachers®

TEST GUIDE

MATHEMATICS SUBTEST III

Subtest Description

This document contains the Mathematics subject matter requirements arranged according to the domains covered by Subtest III of CSET: Mathematics. In parentheses after each named domain is the CCTC-assigned domain code from the Mathematics subject matter requirements.

CS-TG-SD112X-01

California Subject Examinations for Teachers (CSET®)

Mathematics
Subtest III: Calculus; History of Mathematics

Part I: Content Domains for Subject Matter Understanding and Skill in Mathematics

CALCULUS (SMR Domain 5)

Candidates demonstrate an understanding of the trigonometry and calculus contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of trigonometry and calculus and their underlying structures, candidates have a deep conceptual knowledge. They apply the concepts of trigonometry and calculus to solving problems in real-world situations.

0001 Trigonometry (SMR 5.1)

a. Prove that the Pythagorean Theorem is equivalent to the trigonometric identity $\sin^2 x + \cos^2 x = 1$ and that this identity leads to $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$
b. Prove the sine, cosine, and tangent sum formulas for all real values, and derive special applications of the sum formulas (e.g., double angle, half angle)
c. Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems)
d. Know and apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan)
e. Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem)

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 3.0, 14.0, 18.0, 19.0; Algebra II: 24.0, 25.0; Trigonometry: 1.0-6.0, 8.0-11.0, 19.0; Mathematical Analysis: 1.0, 2.0; Calculus: 18.0, 20.0)

0002 Limits and Continuity (SMR 5.2)

a. Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit
b. Show that a polynomial function is continuous at a point

c. Know and apply the Intermediate Value Theorem, using the geometric implications of continuity

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 3.0; Algebra II: 1.0, 15.0; Mathematical Analysis: 8.0; Calculus: 1.0-4.0)

0003 Derivatives and Applications (SMR 5.3)

a. Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative
b. Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change)
c. Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's Theorem, the Mean Value Theorem, and L'Hopital's rule
d. Use the derivative to solve rectilinear motion, related rate, and optimization problems
e. Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity)
f. Solve separable first-order differential equations and apply them to growth and decay problems

(Mathematics Content Standards for California Public Schools, Algebra I: 5.0-8.0, 10.0, 11.0, 13.0, 21.0, 23.0; Geometry: 3.0; Algebra II: 1.0, 9.0, 10.0, 12.0, 15.0; Trigonometry: 7.0, 15.0-19.0; Mathematical Analysis: 5.0, 7.0; Calculus: 1.0, 4.0-12.0, 27.0)

0004 Integrals and Applications (SMR 5.4)

a. Derive definite integrals of standard algebraic functions using the formal definition of integral
b. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums)
c. Prove the Fundamental Theorem of Calculus, and use it to interpret definite integrals as antiderivatives
d. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 9.0; Calculus: 13.0-23.0)

0005 Sequences and Series (SMR 5.5)

a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number)
b. Determine convergence of a given sequence or series using standard techniques (e.g., Ratio, Comparison, Integral Tests)

c. Calculate Taylor series and Taylor polynomials of basic functions

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0, 25.0; Algebra II: 21.0-23.0; Mathematical Analysis: 8.0; Calculus: 23.0-26.0)

HISTORY OF MATHEMATICS (SMR Domain 6)

Candidates understand the chronological and topical development of mathematics and the contributions of historical figures of various times and cultures. Candidates know important mathematical discoveries and their impact on human society and thought. These discoveries form a historical context for the content contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999; e.g., numeration systems, algebra, geometry, calculus).

0006 Chronological and Topical Development of Mathematics (SMR 6.1)

a. Demonstrate understanding of the development of mathematics, its cultural connections, and its contributions to society
b. Demonstrate understanding of the historical development of mathematics, including the contributions of diverse populations as determined by race, ethnicity, culture, geography, and gender

Part II: Subject Matter Skills and Abilities Applicable to the Content Domains in Mathematics

Candidates for Single Subject Teaching Credentials in mathematics use inductive and deductive reasoning to develop, analyze, draw conclusions, and validate conjectures and arguments. As they reason, they use counterexamples, construct proofs using contradictions, and create multiple representations of the same concept. They know the interconnections among mathematical ideas, and use techniques and concepts from different domains and sub-domains to model the same problem. They explain mathematical interconnections with other disciplines. They are able to communicate their mathematical thinking clearly and coherently to others, orally, graphically, and in writing, through the use of precise language and symbols.

Candidates solve routine and complex problems by drawing from a variety of strategies while demonstrating an attitude of persistence and reflection in their approaches. They analyze problems through pattern recognition and the use of analogies. They formulate and prove conjectures, and test conclusions for reasonableness and accuracy. They use counterexamples to disprove conjectures.

Candidates select and use different representational systems (e.g., coordinates, graphs). They understand the usefulness of transformations and symmetry to help analyze and simplify problems. They make mathematical models to analyze mathematical structures in real contexts. They use spatial reasoning to model and solve problems that cross disciplines.

(Mathematics Content Standards for California Public Schools, Grade 6, Mathematical Reasoning: 1.0-3.0; Grade 7, Mathematical Reasoning: 1.0-3.0)