CSET®

California
Subject
Examinations for
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TEST GUIDE

MATHEMATICS
SUBTEST I

Sample Questions and Responses and Scoring Information

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CS-TG-QR110X-03

Sample Test Questions for CSET: Mathematics Subtest I

Below is a set of multiple-choice questions and constructed-response questions that are similar to the questions you will see on Subtest I of CSET: Mathematics. Please note that, as on the actual test form, approximately one third of the multiple-choice questions in this test guide are more complex questions that require 2–3 minutes each to complete. You are encouraged to respond to the questions without looking at the responses provided in the next section. Record your responses on a sheet of paper and compare them with the provided responses.

Note: The use of calculators is not allowed for CSET: Mathematics Subtest I.

Note: In CSET: Mathematics subtests, log *x* represents the base-10 logarithm of *x*.

1. Which of the following statements refutes the claim that $GL_R(3)$, the set of 3 × 3 invertible matrices over the real numbers, is a field?

   A. There exist elements A and B of $GL_R(3)$ such that $AB \neq BA$.

   B. There exist elements A and B of $GL_R(3)$ such that $\det(AB) = \det(A)\det(B)$.

   C. If A is an element of $GL_R(3)$, then there exists a matrix A^{-1} such that $A^{-1}A = I$.

   D. If A is an element of $GL_R(3)$, then there exists a matrix A such that $\det(A) \neq 1$.

2. It is given that the set of complex numbers $\mathbb{C}$ is a commutative ring with a multiplicative identity equal to 1. Let $z_1 = a + bi$ be any nonzero complex number. It can be shown that $\mathbb{C}$ is a field if there exists a $z_2 = x + yi$ satisfying which of the following equations?

   A. $\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

   B. $\begin{bmatrix} -a & b \\ -b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

   C. $\begin{bmatrix} -a & b \\ b & -a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

   D. $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

3. Which of the following sets is an ordered field?

   A. the complex numbers

   B. the rational numbers

   C. the integers

   D. the natural numbers

4. **Use the information below to answer the question that follows.**

   Given $z = a + bi$, $\overline{z} = a - bi$ $(a \neq b \neq 0)$, and $|z^2| = a^2 + b^2$, prove that $z \cdot \overline{z} = |z^2|$.

Step 1	$z \cdot \overline{z} = (a + bi)(a - bi)$
Step 2	$(a + bi)(a - bi) = a^2 - abi + bai + b^2$
Step 3	$a^2 - abi + bai + b^2 = a^2 + b^2$
Step 4	Therefore $z \cdot \overline{z} = |z^2|$

   Which of the following properties is one justification for the simplification made in Step 3?

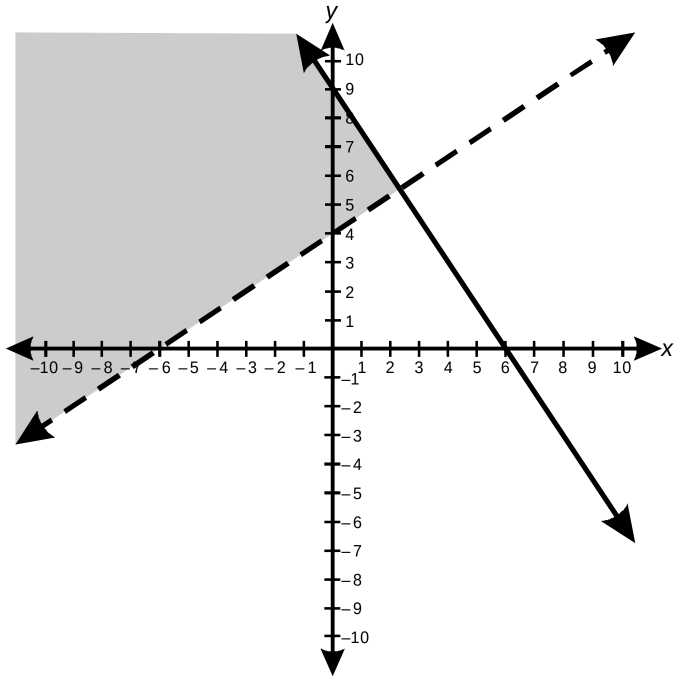
   A. additive inverse property

   B. multiplicative inverse property

   C. multiplicative identity property

   D. modulus property

5. **Use the graph below to answer the question that follows.**



Which of the following systems of inequalities represents the shaded region above?

A. $3y - 2x \geq 12$
$2y + 3x > 18$

B. $3y - 2x \leq 12$
$2y + 3x > 18$

C. $2x - 3y > -12$
$3x + 2y \leq 18$

D. $2x - 3y < -12$
$3x + 2y \leq 18$

6. According to the Rational Root Theorem, which of the following is a possible root of $f(x) = 3x^3 + 7x^2 + 11x + 5$?

   A. –5

   B. –3

   C. $-\frac{3}{5}$

   D. $-\frac{1}{5}$

7. If $f(x)$ is a fourth-degree polynomial with real coefficients such that $\frac{f(x)}{(x - 3)} = q(x) + \frac{8}{(x - 3)}$, which of the following statements about $f(x)$ must be true?

   A. $f(x)$ has a zero at $x = 3$.

   B. The graph of $y = f(x)$ has a local minimum at (–3, 8).

   C. $f(x)$ has two real roots and two complex roots.

   D. The graph of $y = f(x)$ contains the point (3, 8).

8. If $f(x) = -2x^2 + 8x + 16$, then which of the following is the absolute value of the difference between the zeros of f(x)?

   A. 4

   B. $4i$

   C. $4\sqrt{3}$

   D. $4\sqrt{6}$

9. Which of the following are the imaginary parts of the roots of $iz^2 + (2 + i)z + 1$?

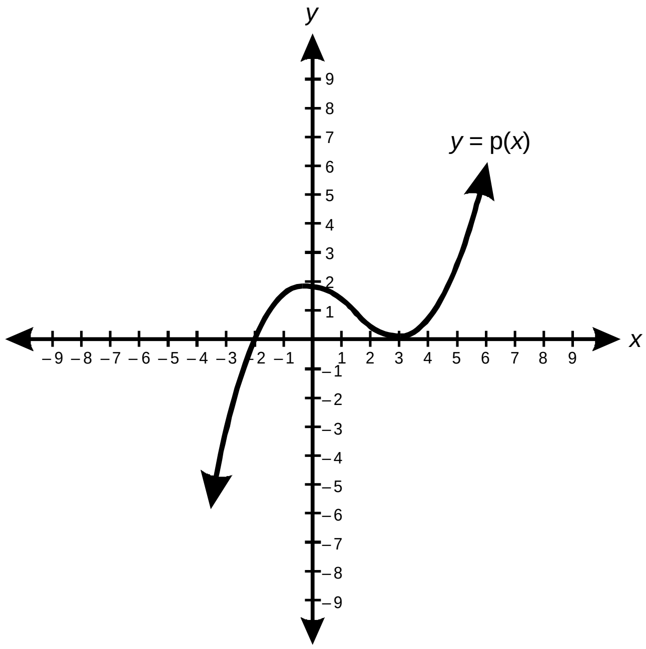
   A. $\frac{-1 \pm \sqrt{3}}{2}$

   B. $\frac{-2 \pm \sqrt{3}}{2}$

   C. $\frac{1 \pm \sqrt{3}}{2}$

   D. $\frac{2 \pm \sqrt{3}}{2}$

10. **Use the graph of a polynomial function below to answer the question that follows.**



Which of the following statements about p(x) must be true?

A. p(x) has at least one complex root

B. p(x) is divisible by (x – 2)

C. p(x) is an odd function

D. p(x) is divisible by $x^2 - 6x + 9$

11. Which of the following is a point of intersection of the function $y = 2\sqrt{x}$ and its inverse function?

A. $\left(\frac{1}{4}, 1\right)$

B. (1, 2)

C. (4, 4)

D. $(8, 4\sqrt{2})$

12. Which of the following is the solution of the equation $2(n - 1) = \frac{d}{\sqrt{L^2 - p^2}}$ in terms of L?

A. $L = \pm\sqrt{p^2 + \left(\frac{d + 1}{2n}\right)^2}$

B. $L = \pm\sqrt{p^2 + \left(\frac{d}{2n} + 1\right)^2}$

C. $L = \pm\sqrt{p^2 + \left(\frac{d}{2(n - 1)}\right)^2}$

D. $L = \pm\sqrt{p^2 + \left(\frac{d^2}{2(n^2 - 1)}\right)^2}$

13. Line ℓ passes through the points (–7, –6) and (8, 14). What is the *x*-intercept of the line that is perpendicular to line ℓ at its *y*-intercept?

A. $\frac{25}{6}$

B. $\frac{13}{3}$

C. $\frac{40}{9}$

D. $\frac{9}{2}$

14. Which of the following is an equation for the slant asymptote of $f(x) = \frac{x^3 - 1}{x^2}$?

A. $y = x - \frac{1}{x^2}$

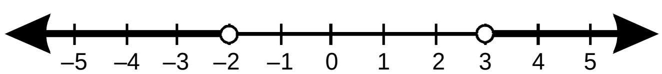
B. $y = x - \frac{1}{x}$

C. $y = x - 1$

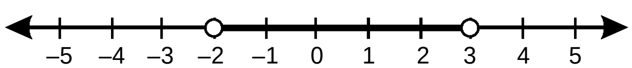
D. $y = x$

15. If $f(x) = x^2 - x - 6$ and $g(x) = \frac{\sqrt{x}}{x}$, which of the following number lines represents the domain of $h(x) = g(f(x))$?

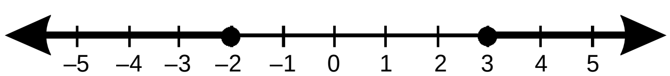
A.



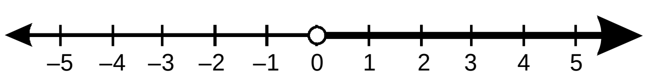
B.



C.



D.



16. If $f(x) = \frac{e^{5x} + 6}{2}$ and $g(f(x)) = x$, then which of the following is equivalent to $g(x)$?

A. $\frac{\ln(2x - 6)}{5}$

B. $\frac{2x - 6}{5}$

C. $\frac{\ln(2x - 6)}{e^5}$

D. $\ln \frac{2x - 6}{5}$

17. The population of a town is given by the function $P(t) = 8800(1.3)^t$ for t in years and $0 \le t \le 8$. Which of the following expresses the number of years until the town reaches a population of 18,000 people?

A. $\frac{\ln 45 - \ln 22}{\ln 1.3}$

B. $\ln 45 - \ln 22 - \ln 1.3$

C. $\frac{\ln 1.3}{\ln 45 - \ln 22}$

D. $\ln \frac{45}{22} - \ln 1.3$

18. Given any two vectors $\vec{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}}$ such that $|\vec{\boldsymbol{a}}| = |\vec{\boldsymbol{b}}| = 1$, which of the following statements about the inner product, $\vec{\boldsymbol{a}} \bullet \vec{\boldsymbol{b}}$, must be true?

    A. $\vec{\boldsymbol{a}} \bullet \vec{\boldsymbol{b}} = 1$

    B. $-1 \le \vec{\boldsymbol{a}} \bullet \vec{\boldsymbol{b}} \le 1$

    C. $1 \le \vec{\boldsymbol{a}} \bullet \vec{\boldsymbol{b}} \le \sqrt{2}$

    D. $1 \le \vec{\boldsymbol{a}} \bullet \vec{\boldsymbol{b}} \le 2$

19. Given vectors $\vec{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}}$ such that $|\vec{\boldsymbol{a}}| = \frac{10\sqrt{3}}{3}$, $|\vec{\boldsymbol{b}}| = 2$, and $\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}} = 2\sqrt{5}\,\vec{\boldsymbol{i}} + 2\sqrt{5}\,\vec{\boldsymbol{j}} - 2\sqrt{15}\,\vec{\boldsymbol{k}}$, what is the angle between $\vec{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}}$?

    A. 30°

    B. 45°

    C. 60°

    D. 90°

20. **Use the matrix below to answer the question that follows.**

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Which of the following matrices has the same determinant as the matrix above?

A. $\begin{bmatrix} -2 & -3 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & 2 \\ 2 & 4 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 4 & 6 \\ -2 & -3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 1 & 3 \\ -3 & -2 & 2 \\ 2 & 1 & 1 \end{bmatrix}$

21. Given the equation $\det(A - \lambda I) = 0$, where $A = \begin{bmatrix} 3 & 1 \\ -2 & 6 \end{bmatrix}$ and I is the identity matrix, which of the following is a value of λ?

A. -4

B. $-\frac{1}{3}$

C. 5

D. 20

22. If $A\vec{\boldsymbol{x}} = \vec{\boldsymbol{b}}$ represents a linear system of equations in three-dimensional space and the det(A) $\neq 0$, then the solution to this system is a:

    A. point.

    B. line.

    C. two-dimensional plane.

    D. three-dimensional plane.

23. In order to identify all the prime numbers less than 200, a person writes the numbers from 1 to 200, and eliminates all the multiples of 2, then all the multiples of 3. To complete this task, the person will have to eliminate the multiples of which additional numbers?

    A. 5, 7, 9, 11

    B. 7, 9, 11, 13

    C. 5, 7, 11, 13

    D. 7, 11, 13, 17

24. If x, y, and z are nonnegative integers, what is the total number of factors of $2^x 3^y 5^z$?

A. $(2 + 3 + 5)(x + y + z)$

B. xyz

C. $(x + 1)(y + 1)(z + 1)$

D. $x^2 y^3 z^5$

25. **Use the theorem about integers below to answer the question that follows.**

> Given that $a > b$, $a = nb + r$, and that d divides a and b, then d divides r.

Which of the following describes why this statement is true?

A. If d divides a and b, then d divides $a - b$.

B. If d divides a and b, then d divides n.

C. If d divides a and b, then d divides ab.

D. If d divides a and b, then d is the greatest common divisor of a and b.

26. If a, b, n, and r are positive integers such that $a > b$, and $a = nb + r$, then the greatest common divisor of a and b (gcd(a, b)) is also equal to which of following?

A. gcd(a, n)

B. gcd(b, r)

C. gcd(n, r)

D. gcd(n, b)

27. **Use the informal proof below to answer the question that follows.**

> Let x be a positive integer that is not a perfect square. Assume that $\sqrt{x} = \frac{y}{z}$, and that y and z are relatively prime integers. It follows that $x = \left(\frac{y}{z}\right)^2 = \frac{y^2}{z^2}$. Since x is an integer, z^2 is equal to 1, and x is the square of y. But this is not possible.

Which of the following is shown by this informal proof?

A. The square root of any integer that is not a perfect square is an algebraic number.

B. The square of any rational number is also a rational number.

C. The square root of any positive integer that is not a perfect square is irrational.

D. The square of any real number is also a real number.

28. **Complete the exercise that follows.**

Show that the subset of complex numbers of the form $a + bi$ with a and b rational numbers satisfies the axioms of a field under the operations of addition and multiplication of complex numbers.

29. **Complete the exercise that follows.**

A cubic function of the form $f(x) = x^3 + c$, where c is a real number, has one zero at $x = 1 + i\sqrt{3}$.

- Find the cubic function; and
- sketch the graph of the function and label any intercepts.

30. **Complete the exercise that follows.**

If vectors $\vec{\boldsymbol{a}} = (a_1, a_2)$ and $\vec{\boldsymbol{b}} = (b_1, b_2)$ are perpendicular, $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, and if we identify vectors with column matrices in the usual manner, then show that the vectors $A\vec{\boldsymbol{a}}$ and $A\vec{\boldsymbol{b}}$ are perpendicular for all values of θ.

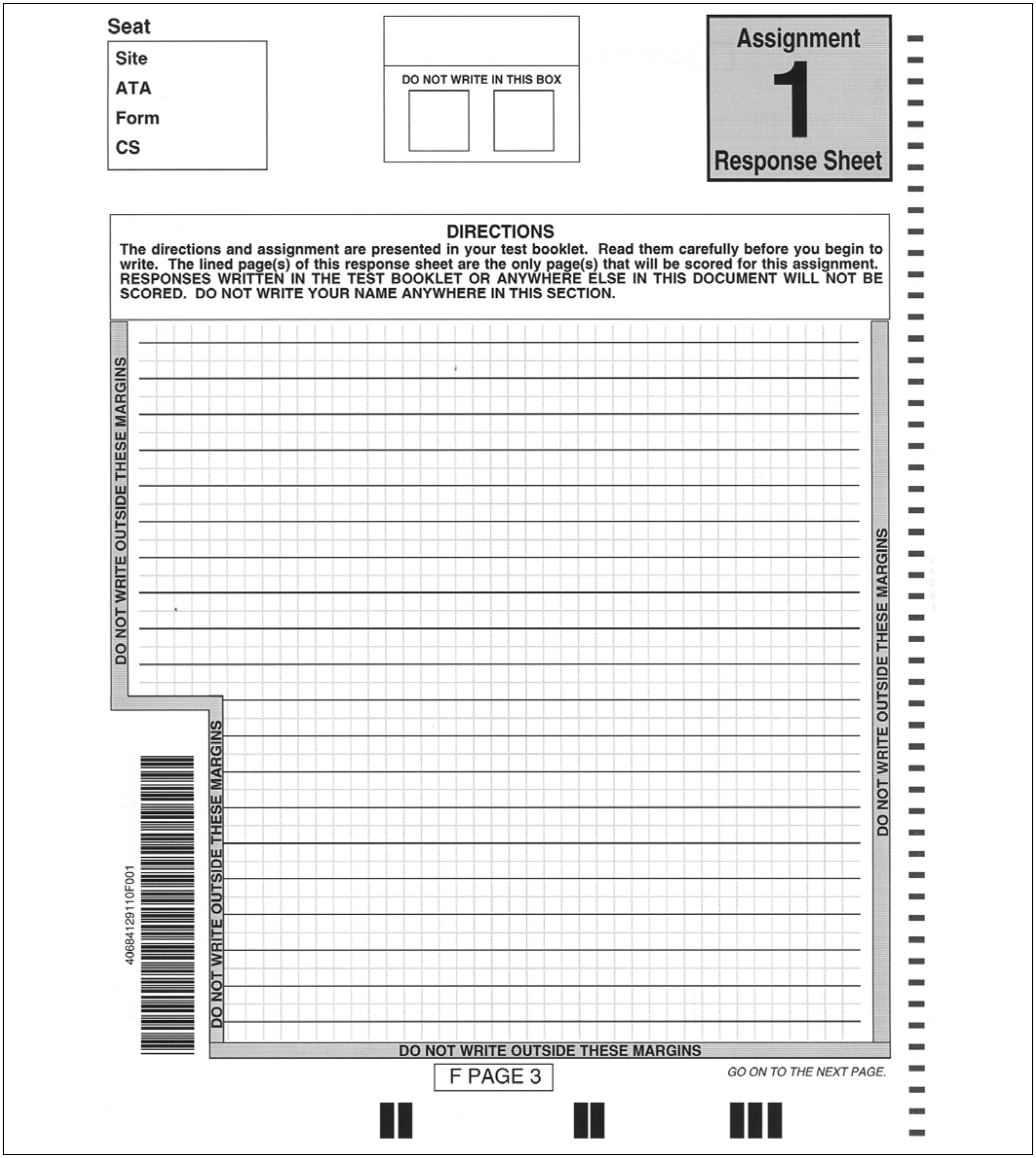
31. **Complete the exercise that follows.**

Use the principle of mathematical induction to prove the following statement.

$$\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \, . \, . \, . + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

Sample Written Response Sheets for CSET: Mathematics Subtest I

For questions 28–31, examinees would record their written response to each question on a two-page response sheet located in their answer document. The length of their response to each question is limited to the lined space available on the response sheet. A sample of the response sheet is provided below.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DO NOT WRITE IN THIS BOX

Assignment 1 Response Sheet

DIRECTIONS

The directions and assignment are presented in your test booklet. Read them carefully before you begin to write. The lined page(s) of this response sheet are the only page(s) that will be scored for this assignment. RESPONSES WRITTEN IN THE TEST BOOKLET OR ANYWHERE ELSE IN THIS DOCUMENT WILL NOT BE SCORED. DO NOT WRITE YOUR NAME ANYWHERE IN THIS SECTION.

DO NOT WRITE OUTSIDE THESE MARGINS

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F PAGE 3

GO ON TO THE NEXT PAGE.

CONTINUE YOUR RESPONSE HERE

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F PAGE 4

STOP. END OF ASSIGNMENT 1.

Annotated Responses to Sample Multiple-Choice Questions for CSET: Mathematics Subtest I

Algebra

1. **Correct Response: A.** (SMR Code: 1.1) A field is a commutative ring with identity in which every nonzero element has an inverse. The set of 3×3 invertible matrices is not a commutative ring because there exist matrices A and B such that $AB \neq BA$.

2. **Correct Response: D.** (SMR Code: 1.1) Since it is given that $\mathbb{C}$ is a commutative ring, to show that it is a field only requires showing that any nonzero $z_1 = a + bi$ has a multiplicative inverse in $\mathbb{C}$. Let $z_2 = x + yi$ be the multiplicative inverse of z_1. Then $z_1 z_2 = 1 + 0i = (a + bi)(x + yi) = ax + ayi + bxi - by = ax - by + (bx + ay)i = 1 + 0i$. The system of equations that results is $ax - by = 1$ and $bx + ay = 0$. The matrix representation of this system is $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

3. **Correct Response: B.** (SMR Code: 1.1) Of the sets given in the response choices, only the complex numbers and the rational numbers are fields. Since the complex numbers are not ordered, response choice B is the correct response.

4. **Correct Response: A.** (SMR Code: 1.1) Note that in the expression on the left-hand side of Step 3, $a^2 - abi + bai + b^2$, the two terms $-abi + bai$ sum to zero, by the additive inverse property.

5. **Correct Response: D.** (SMR Code: 1.2) The equation of the solid line is $y = -\frac{3}{2}x + 9$, or $2y + 3x = 18$ in standard form. Substituting a point in the shaded region [(0, 6), for example] shows that the inequality is $2y + 3x \leq 18$. The dotted line is given by $3y - 2x = 12$. Substituting (0, 6) gives the inequality $3y - 2x > 12$ (the dotted line indicates that points on the boundary line are not solutions to the inequality). This inequality can be rewritten as $-3y + 2x < -12$, so response choice D is correct.

6. **Correct Response: A.** (SMR Code: 1.2) By the Rational Root Theorem, possible roots for $f(x)$ are factors of 5 divided by factors of 3. These are $\pm\frac{1}{3}$, $\pm\frac{5}{3}$, ± 1, and ± 5. Of the available choices in the response options, A, or -5, is the correct response.

7. **Correct Response: D.** (SMR Code: 1.3) Solve the given equation for $f(x)$ to obtain $f(x) = q(x)(x - 3) + 8$. Let $x = 3$, so that $f(3) = q(3)(3 - 3) + 8 \Rightarrow f(3) = 8$. Therefore, the point (3, 8) is on the graph of $y = f(x)$ and the correct response is D.

8. **Correct Response: C.** (SMR Code: 1.2) To find the zeros of f(x), set the function equal to zero and solve for x by using the quadratic formula: $-2x^2 + 8x + 16 = 0 \Rightarrow x = \frac{-8 \pm \sqrt{8^2 - 4(-2)(16)}}{2(-2)} \Rightarrow x = \frac{-8 \pm \sqrt{192}}{-4}$. Simplifying $\sqrt{192}$ gives $x = \frac{-8 \pm 8\sqrt{3}}{-4} = 2 \pm 2\sqrt{3}$. The absolute value of the difference between the zeros is $\left|\left(2 + 2\sqrt{3}\right) - \left(2 - 2\sqrt{3}\right)\right|$, which simplifies to $\left|4\sqrt{3}\right|$ or $4\sqrt{3}$.

9. **Correct Response: D.** (SMR Code: 1.2) Finding the roots of $iz^2 + (2 + i)z + 1$ involves setting the expression equal to 0 and solving for z. The quadratic formula gives $z = \frac{-(2 + i) \pm \sqrt{(2 + i)^2 - 4(i)(1)}}{2(i)} \Rightarrow z = \frac{-2 - i \pm \sqrt{3}}{2i} \Rightarrow z = i - \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$. Since the roots of the equation are $-\frac{1}{2} + \left(1 \pm \frac{\sqrt{3}}{2}\right)i$, their imaginary parts are $\frac{2 \pm \sqrt{3}}{2}$.

10. **Correct Response: D.** (SMR Code: 1.2) The polynomial p(x) has x-intercepts at 3 and –2, so $(x - 3)$ and $(x + 2)$ are factors. In fact, $x = 3$ is a double root, since the function is tangent to the x-axis at $x = 3$. This means that $(x - 3)^2$, or $(x^2 - 6x + 9)$, is a factor of p(x), so p(x) is divisible by $x^2 - 6x + 9$.

11. **Correct Response: C.** (SMR Code: 1.3) The inverse of $y = 2\sqrt{x}$ is $x = 2\sqrt{y}$, which can be rewritten as $y = \frac{x^2}{4}$. However, domain restrictions limit this inverse to only positive x-values, so the inverse of $y = 2\sqrt{x}$ is $y = \frac{x^2}{4}, x \geq 0$. To find the intersection of these two functions, solve $2\sqrt{x} = \frac{x^2}{4}$ for x values ≥ 0. $4x = \frac{x^4}{16} \Rightarrow x^4 - 64x = 0 \Rightarrow x(x^3 - 64) = 0$. Hence $x = 0$ or $x = 4$. Since $y = 2\sqrt{x}$, (0, 0) and (4, 4) are points where the two curves intersect. Therefore C is the correct response.

12. **Correct Response: C.** (SMR Code: 1.3) To solve for L, first isolate $\sqrt{L^2 - p^2}$ by multiplying both sides of the equation by $\frac{\sqrt{L^2 - p^2}}{2(n - 1)}$. This gives $\sqrt{L^2 - p^2} = \frac{d}{2(n - 1)}$. Then $L^2 - p^2 = \left(\frac{d}{2(n - 1)}\right)^2 \Rightarrow L^2 = p^2 + \left(\frac{d}{2(n - 1)}\right)^2$. Solving for L requires taking the square root of both sides, so $L = \pm\sqrt{p^2 + \left(\frac{d}{2(n - 1)}\right)^2}$.

13. **Correct Response: C.** (SMR Code: 1.3) The slope of line ℓ is $\frac{-6 - 14}{-7 - 8}$, or $\frac{4}{3}$. Substituting one of the given points into $y = \frac{4}{3}x + b$ gives an equation of $y = \frac{4}{3}x + \frac{10}{3}$, so the y-intercept of ℓ is $\left(0, \frac{10}{3}\right)$. Therefore, a line perpendicular to ℓ at its y-intercept has an equation of $y = -\frac{3}{4}x + \frac{10}{3}$. To find the x-intercept of this line, substitute 0 for y and solve for x. The x-intercept is $\frac{40}{9}$.

14. **Correct Response: D.** (SMR Code: 1.3) Note that $f(x) = \frac{x^3 - 1}{x^2} = x - \frac{1}{x^2}$. As x approaches positive or negative infinity, $\frac{1}{x^2}$ approaches 0 and $f(x)$ approaches x. Therefore, the asymptote of $f(x)$ is the line $y = x$.

15. **Correct Response: A.** *(*SMR Code: 1.3) The composition of $g(f(x)) = h(x) = \frac{\sqrt{x^2 - x - 6}}{x^2 - x - 6} = \frac{1}{\sqrt{x^2 - x - 6}}$. The domain of $h(x)$ is such that $x^2 - x - 6 > 0$. Since $x^2 - x - 6 = (x + 2)(x - 3)$, $x = -2$ and $x = 3$ are zeros of this expression. This expression will be positive when both $(x + 2)$ and $(x - 3)$ are positive or when both $(x + 2)$ and $(x - 3)$ are negative. This occurs when $x < -2$ or $x > 3$, as shown in the graph in response A.

16. **Correct Response: A.** (SMR Code: 1.3) If $g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverses of each other. If $f(x) = \frac{e^{5x} + 6}{2}$, then $x = \frac{e^{5g(x)} + 6}{2}$. This equation can be rewritten as $2x - 6 = e^{5g(x)}$, and taking the natural log of both sides yields $\ln(2x - 6) = \ln e^{5g(x)} \Rightarrow 5g(x)(\ln e) = \ln(2x - 6)$. Then, since $\ln e = 1$, $g(x) = \frac{1}{5}\ln(2x - 6)$.

17. **Correct Response: A.** (SMR Code: 1.3) To find when the population equals 18,000 people, solve $P(t)$ for t. If $P(t) = 18{,}000$ and $P(t) = 8800(1.3)^t$, then $18{,}000 = 8800(1.3)^t \Rightarrow \frac{18000}{8800} = (1.3)^t \Rightarrow \frac{45}{22} = (1.3)^t$. Therefore, $\ln\frac{45}{22} = \ln(1.3)^t$ and $t = \frac{\ln 45 - \ln 22}{\ln 1.3}$.

18. **Correct Response: B.** (SMR Code: 1.4) The dot product of vectors $\vec{a}$ and $\vec{b}$ can be written as $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$, where θ is the angle between the two vectors. Given that $\vec{a}$ and $\vec{b}$ are unit vectors, $\vec{a} \cdot \vec{b} = \cos\theta$. Since $-1 \le \cos\theta \le 1$, $-1 \le \vec{a} \cdot \vec{b} \le 1$.

19. **Correct Response: C.** (SMR Code: 1.4) The magnitude of the cross product of two vectors is given by $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$. Since $|\vec{a}|$ and $|\vec{b}|$ are both given and $|\vec{a} \times \vec{b}| = \sqrt{(2\sqrt{5})^2 + (2\sqrt{5})^2 + (-2\sqrt{15})^2} = 10$, it follows that $= \left(\frac{10\sqrt{3}}{3}\right)(2)\sin\theta = 10 \Rightarrow \frac{\sqrt{3}}{2} = \sin\theta$. Since $\arcsin\frac{\sqrt{3}}{2} = 60°$, the angle between the two vectors is 60°.

20. **Correct Response: B.** (SMR Code: 1.4) The matrix in each response choice is obtained from the given matrix by an elementary row operation. The matrix in response choice B is obtained from the given matrix by adding row 1 to row 3. This elementary row operation does not change the value of the determinant of the matrix.

21. **Correct Response: C.** (SMR Code: 1.4) Note that $(A - \lambda I) = \begin{bmatrix} 3 & 1 \\ -2 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 3-\lambda & 1 \\ -2 & 6-\lambda \end{bmatrix}$. Therefore, $\det(A - \lambda I) = \det \begin{bmatrix} 3-\lambda & 1 \\ -2 & 6-\lambda \end{bmatrix} = (3-\lambda)(6-\lambda) - (-2) = \lambda^2 - 9\lambda + 20$. Since $\det(A - \lambda I) = 0$, then $0 = \lambda^2 - 9\lambda + 20$. Solving for λ gives $0 = (\lambda - 5)(\lambda - 4)$ and $\lambda = 4$ or $\lambda = 5$. Of the available responses, C is correct.

22. **Correct Response: A.** (SMR Code: 1.4) Since $\det(A) \neq 0$, then A^{-1} exists and is unique. Thus, $A^{-1}A\vec{\boldsymbol{x}} = A^{-1}\vec{\boldsymbol{b}} \Rightarrow I\vec{\boldsymbol{x}} = A^{-1}\vec{\boldsymbol{b}} \Rightarrow \vec{\boldsymbol{x}} = A^{-1}\vec{\boldsymbol{b}}$ is a unique solution to the linear system of equations given by $A\vec{\boldsymbol{x}} = \vec{\boldsymbol{b}}$. Since this is a linear system in three-dimensional space, $\vec{\boldsymbol{x}}$ represents a point in space.

Number Theory

23. **Correct Response: C.** (SMR Code: 3.1) To identify all prime numbers less than 200, it is only necessary to eliminate all multiples of each prime number less than (or equal to) $\sqrt{200}$. This is because no composite number less than 200 will have a factor greater than $\sqrt{200}$ without having another factor less than $\sqrt{200}$. Since $\sqrt{200} \approx 14.14$, the person would need to check multiples of 2, 3, 5, 7, 11 and 13.

24. **Correct Response: C.** (SMR Code: 3.1) Each factor of $2^x3^y5^z$ is the product of between 0 and x twos, between 0 and y threes, and between 0 and z fives. Since this yields $(x + 1)$ possible products of twos, $(y + 1)$ possible products of threes and $(z + 1)$ possible products of fives, there are $(x + 1)(y + 1)(z + 1)$ factors.

25. **Correct Response: A.** (SMR Code: 3.1) The proof begins by assuming that d divides a and b. The object is to prove that d divides r. Since $a = nb + r$, then $r = a - nb$. If d divides a and b, then d divides $a - b$. If d divides $a - b$, then d divides $a - 2b$. Likewise, d divides $a - 3b$ and so on. Since d divides $a - nb$, d divides r. Thus, the correct response is A.

26. **Correct Response: B.** (SMR Code: 3.1) Since $a = nb + r$, then $r = a - nb$ and any number that divides a and b must also divide r. This includes the greatest common divisor of a and b, so $\gcd(a, b)$ divides r. By definition, the greatest common divisor of a and b divides b, so $\gcd(a, b)$ also divides b. Therefore, $\gcd(a, b)$ divides both b and r. This divisor is either the greatest common divisor of b and r, or less than the greatest common divisor of b and r, so the following holds: $\gcd(a, b) \leq \gcd(b, r)$. Similarly, any number that divides both b and r must also divide a, so $\gcd(b, r)$ divides a. Also, $\gcd(b, r)$ divides b. Therefore, $\gcd(b, r)$ divides both a and b and it follows that $\gcd(b, r) \leq \gcd(a, b)$. Combining the two inequalities shows that $\gcd(b, r) = \gcd(a, b)$.

27. **Correct Response: C.** (SMR Code: 3.1) The proof begins by assuming that the square root of a positive integer that is not a perfect square is rational, by rewriting it as a fraction. This assertion leads to a contradiction, which shows that the initial assumption is not correct.

Examples of Responses to Sample Constructed-Response Questions for CSET: Mathematics Subtest I

Algebra

Question #28 (Score Point 4 Response)

Since the complex numbers are a field, they obey the commutative and associative properties of addition and multiplication, and the distributive property of multiplication over addition.

∴ numbers of the form $a + bi$, with a and b rational numbers, obey these properties. Hence, we need only show that numbers of this form are closed under addition, multiplication, and inverses.

For example, adding or multiplying 2 complex numbers of this form results in a complex number with real and imaginary parts that are rational.

Let $a + bi$ and $c + di$ be two complex numbers with a, b, c and d all rational.

Closed under addition: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Since the sum of two rational numbers is a rational number, $a + c$ and $b + d$ are both rational numbers.

∴ the sum of two complex numbers with rational real and imaginary parts $(a, c$ + $b, d)$ is a complex number with rational real and imaginary parts (i.e., $a + c$ and $b + d$, resp.).

continued on next page

Question #28 (Score Point 4 Response) ***continued***

Additive identity: $0 + 0i$ is the additive identity, since

$(a + bi) + (0 + 0i) = (a + 0) + (b + 0)i = a + bi$

and 0 is a rational number.

Additive inverse: For any $a + bi$, $-a - bi$ is its additive inverse.

$\Rightarrow a + bi + (-a - bi) = (a + -a) + (b + -b)i = 0 + 0i$

Since a and b are rational, $-a$ and $-b$ are rational.

Closed under multiplication: $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Since the product, difference, and sum of two rational numbers are each a rational number, $ac - bd$ and $ad + bc$ are both rational numbers.

$\therefore$ product of two complex numbers with rational real and imaginary parts (i.e., a, c + b, d) is a complex number with rational real and imaginary parts (i.e., $ac - bd$ + $ad + bc$, resp.).

Multiplicative identity: $1 = 1 + 0i$ is the multiplicative identity, since

$(a + bi)(1 + 0i) = a + bi + a \cdot 0i - b \cdot 0 = a + bi + 0i - 0 = (a - 0) + (b + 0)i$

$= a + bi$

Both 0 and 1 are rational numbers.

Multiplicative inverse: The multiplicative inverse can be found as follows:

$(a + bi)(a - bi) = a^2 + abi - abi + b^2 = a^2 + b^2$.

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Question #28 (Score Point 4 Response) ***continued***

Since the square of a rational number is a rational, a^2 and b^2 are both rationals, and so $a^2 + b^2$ is a rational number.

If a and $b \neq 0$, then $\frac{(a + bi)(a - bi)}{a^2 + b^2} = 1$.

Hence the multiplicative inverse of $a + bi$ is $\frac{a - bi}{a^2 + b^2}$, which can be written as

$\frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i$.

Since the square, sum, and division of two rational numbers are each a rational number, $\frac{a}{a^2 + b^2}$ and $\frac{b}{a^2 + b^2}$ are both rational numbers.

Therefore, the set of numbers of the form $a + bi$, with a and b rational, satisfies the axioms of a field under addition and multiplication.

Question #28 (Score Point 3 Response)

Let $z = a + bi$ and $w = c + di$ be two complex numbers with a, b, c and d rational.

Closure:

$(a + bi) + (c + di) = (a + c) + (b + d)i$

Since the sum of two rationals is a rational, $a + c$ and $b + d$ are both rationals.

$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Since the product, difference, and sum of two rationals are each rational, $ac - bd$ and $ad + bc$ are both rationals.

$\therefore$ the sum and the product of two complex numbers with rational real and imaginary parts is a complex number with rational real and imaginary parts.

Identity:

$(a + bi) + (0 + 0i) = (a + 0) + (b + 0)i = a + bi$

So, $0 + 0i$ is the additive identity of $a + bi$.

$$(a + bi)(1 + 0i) = a + bi + a \cdot 0i - b \cdot 0 = a + bi + 0i - 0 = (a - 0) + (b + 0)i$$
$$= a + bi$$

So, $1 + 0i = 1$ is the multiplicative identity, where 0 and 1 are rational numbers.

Hence the subset of numbers of the form $z = a + bi$, with a and b rational, satisifes the axioms of a field.

Question #28 (Score Point 2 Response)

$(a + bi) + (c + di) = (a + c) + (b + d)i$

$(a + bi) - (c + di) = (a - c) + (b - d)i$

$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

So adding, subtracting, and multiplying complex numbers of the form $a + bi$ results in a complex number of the same form.

Question #28 (Score Point 1 Response)

Field Properties are: Commutative, Associative, and Distributive.

Commutative: $(3 + 2i) + (4 + 5i) = (4 + 5i) + (3 + 2i)$

$(3 + 2i)(4 + 5i) = (4 + 5i)(3 + 2i)$

Associative: $[(3 + 2i) + (4 + 5i)] + (2 + 3i) = (3 + 2i) + [(4 + 5i) + (2 + 3i)]$

$[(3 + 2i)(4 + 5i)] (2 + 3i) = (3 + 2i) [(4 + 5i)(2 + 3i)]$

Distributive: $[(3 + 2i) + (4 + 5i)] (2 + 3i) = (3 + 2i) (2 + 3i) + (4 + 5i)(2 + 3i)$

Question #29 (Score Point 4 Response)

If $f(x) = x^3 + c$, where c is a real number, and f(x) has a zero at $x = 1 + i\sqrt{3}$, then its conjugate $1 - i\sqrt{3}$ must also be a root.

Thus $x - (1 + i\sqrt{3})$ and $x - (1 - i\sqrt{3})$ both are factors of $f(x) = x^3 + c$, so their product is a quadratic factor of $f(x) = x^3 + c$.

$$[x - (1 + i\sqrt{3})][x - (1 - i\sqrt{3})] = [x - 1 - i\sqrt{3}][x - 1 + i\sqrt{3}]$$

$$= x^2 - x + xi\sqrt{3} - x + 1 - i\sqrt{3} - xi\sqrt{3} + i\sqrt{3} - 3i^2$$

$$= x^2 - 2x + 4$$ is a quadratic factor of $f(x) = x^3 + c$

Let $a = \sqrt[3]{c}$, so $f(x) = x^3 + a^3$.

Now factor $x^3 + a^3$ as the sum of 2 cubes:

$$f(x) = x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

$$= (x + a)(x^2 - 2x + 4),$$ since $x^2 - 2x + 4$ is a factor of $x^3 + c$.

Thus $a = 2 \Rightarrow f(x) = x^3 + a^3 = x^3 + 2^3 = x^3 + 8$

x	f(x)
0	8
−2	0

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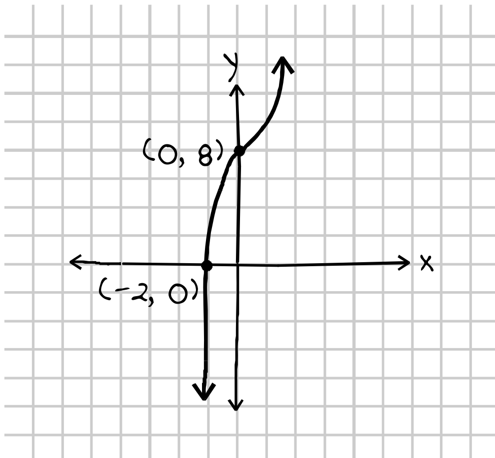
Question #29 (Score Point 4 Response) ***continued***

The graph of $f(x) = x^3 + 8$ is the graph of $f(x) = x^3$ shifted up 8 units.



Intercepts are (0, 8) and (−2, 0).

Question #29 (Score Point 3 Response)

$$\left(1 + i\sqrt{3}\right)^3 + c = 0$$

$$\left(1 + i\sqrt{3}\right)^3 = -c$$

$$\left(1 + i\sqrt{3}\right)^2\left(1 + i\sqrt{3}\right) = -c$$

$$\left(1 + 2i\sqrt{3} + i^2 \cdot 3\right)\left(1 + i\sqrt{3}\right) = -c$$

$$\left(-2 + 2i\sqrt{3}\right)\left(1 + i\sqrt{3}\right) = -c$$

$$-2 - 2i\sqrt{3} + 2i\sqrt{3} + 2 \cdot 3 \cdot i^2 = -c$$

$$c = 8$$

$$f(x) = x^3 + 8$$

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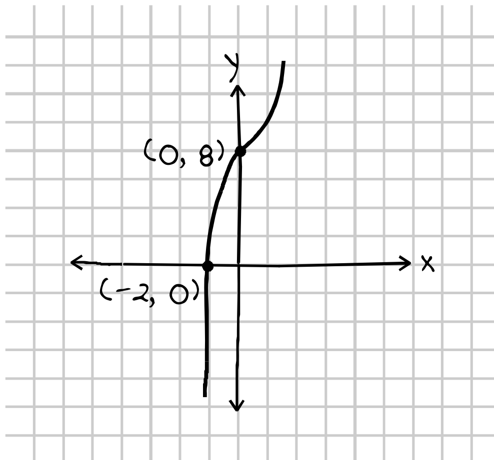
Question #29 (Score Point 3 Response) ***continued***



Intercepts are (0, 8) and (−2, 0).

Question #29 (Score Point 2 Response)

Since one root is $1 + i\sqrt{3}$, there must be another root $1 - i\sqrt{3}$.

$\left(1 + i\sqrt{3}\right)\left(1 - i\sqrt{3}\right) + c = 0$

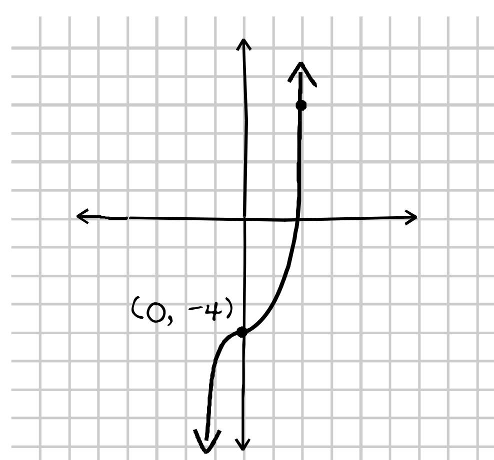
$1 - i\sqrt{3} + i\sqrt{3} - 3i^2 = -c$

$4 = -c$

$c = -4$

$f(x) = x^3 - 4$

x	$x^3 - 4$	y
0	$0 - 4$	-4
1	$1 - 4$	-3
-1	$-1 - 4$	-5
2	$8 - 4$	4
-2	$-8 - 4$	-12

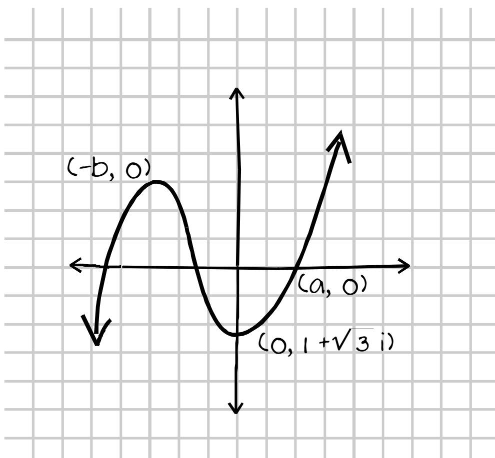


Question #29 (Score Point 1 Response)

$x^3 + c$ is a cubic function so it should have 3 roots. Since one root is imaginary, there must be 2 real roots.

$$\left[x - (1 + \sqrt{3}i)\right](x - a)(x - b) = 0$$

simplify?



Question #30 (Score Point 4 Response)

The dot product $\vec{a} \bullet \vec{b} = a_1b_1 + a_2b_2 = 0$ if and only if the vectors $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$ are perpendicular. Want to show that $A\vec{a} \bullet B\vec{b} = 0$.

$$A\vec{a} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} a_1\cos\theta - a_2\sin\theta \\ a_1\sin\theta + a_2\cos\theta \end{bmatrix}$$

$$A\vec{b} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_1\cos\theta - b_2\sin\theta \\ b_1\sin\theta + b_2\cos\theta \end{bmatrix}$$

$$A\vec{a} \bullet A\vec{b} = (a_1\cos\theta - a_2\sin\theta, a_1\sin\theta + a_2\cos\theta) \bullet (b_1\cos\theta - b_2\sin\theta, b_1\sin\theta + b_2\cos\theta)$$

$$= a_1b_1\cos^2\theta - a_1b_2\cos\theta\sin\theta - a_2b_1\cos\theta\sin\theta + a_2b_2\sin^2\theta +$$

$$a_1b_1\sin^2\theta + a_1b_2\cos\theta\sin\theta + a_2b_1\cos\theta\sin\theta + a_2b_2\cos^2\theta$$

$$= a_1b_1\cos^2\theta + a_1b_1\sin^2\theta + a_2b_2\sin^2\theta + a_2b_2\cos^2\theta$$

$$= a_1b_1(\cos^2\theta + \sin^2\theta) + a_2b_2(\cos^2\theta + \sin^2\theta)$$

Since $\cos^2\theta + \sin^2\theta = 1$, $A\vec{a} \bullet A\vec{b} = a_1b_1 + a_2b_2 = 0$ (since $\vec{a}$ and $\vec{b}$ are perpendicular).

Hence the vectors $A\vec{a}$ and $A\vec{b}$ are perpendicular for all θ.

Question #30 (Score Point 3 Response)

$\vec{a} \bullet \vec{b} = 0$ since $\vec{a}$ and $\vec{b}$ are perpendicular.

$$A\vec{a} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} a_1\cos\theta - a_2\sin\theta \\ a_1\sin\theta + a_2\cos\theta \end{bmatrix}$$

$$A\vec{b} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_1\cos\theta - b_2\sin\theta \\ b_1\sin\theta + b_2\cos\theta \end{bmatrix}$$

$A\vec{a} \bullet A\vec{b}$

$$= (a_1\cos\theta - a_2\sin\theta,\ a_1\sin\theta + a_2\cos\theta) \bullet (b_1\cos\theta - b_2\sin\theta,\ b_1\sin\theta - b_2\cos\theta)$$

$$= a_1b_1\cos^2\theta - a_1b_2\cos\theta\sin\theta - a_2b_1\cos\theta\sin\theta + a_2b_2\sin^2\theta +$$

$$a_1b_1\sin^2\theta + a_1b_2\cos\theta\sin\theta + a_2b_1\cos\theta\sin\theta + a_2b_2\cos^2\theta$$

$$= a_1b_1\cos^2\theta + a_1b_1\sin^2\theta + a_2b_2\sin^2\theta + a_2b_2\cos^2\theta$$

$$= a_1b_1(\sin^2\theta + \cos^2\theta) + a_2b_2(\sin^2\theta + \cos^2\theta)$$

$$= 0$$

Therefore $A\vec{a}$ and $A\vec{b}$ are perpendicular.

Question #30 (Score Point 2 Response)

If $A\vec{a} \bullet A\vec{b} = 0$, then $A\vec{a}$ and $A\vec{b}$ are perpendicular.

$$A\vec{a} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} a_1\cos\theta - a_2\sin\theta \\ a_1\sin\theta + a_2\cos\theta \end{bmatrix}$$

$$A\vec{b} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_1\cos\theta - b_2\sin\theta \\ b_1\sin\theta + b_2\cos\theta \end{bmatrix}$$

$A\vec{a} \bullet A\vec{b}$

$$= (a_1\cos\theta - a_2\sin\theta,\ a_1\sin\theta + a_2\cos\theta) \bullet (b_1\cos\theta - b_2\sin\theta,\ b_1\sin\theta - b_2\cos\theta)$$

$$= a_1b_1\cos^2\theta + a_2b_2\sin^2\theta + a_1b_1\sin^2\theta + a_2b_2\cos^2\theta$$

$$= \cos^2\theta\,(a_1b_1 + a_2b_2) + \sin^2\theta\,(a_2b_2 + a_1b_1)$$

So $A\vec{a} \bullet A\vec{b} = 0$ if $a_1b_1 + a_2b_2 = 0$

Question #30 (Score Point 1 Response)

If two vectors are perpendicular, then $a \bullet b = 0$

$$Aa = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$Bb = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$Aa \bullet Bb = 0.$

Number Theory

Question #31 (Score Point 4 Response)

Initial Step: show the statement is true for $n = 1$.

left-hand side: $\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{n(n+1)} = \frac{1}{1(1+1)} = \frac{1}{2}$

right-hand side: $\frac{n}{n+1} = \frac{1}{1+1} = \frac{1}{2}$

$\therefore$ the statement is true for $n = 1$.

Inductive Step: assume the statement is true for $n = K$, and then prove that it is true for $n = K + 1$.

So assume $\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{K(K+1)} = \frac{K}{K+1}$ (1)

Now want to show that $\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{(K+1)(K+2)} = \frac{K+1}{K+2}$

Adding $\frac{1}{(K+1)(K+2)}$ to each side of equation (1) gives:

$$\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)} = \frac{K}{K+1} + \frac{1}{(K+1)(K+2)}$$

$$= \frac{K(K+2)+1}{(K+1)(K+2)}$$

$$= \frac{K^2 + 2K + 1}{(K+1)(K+2)}$$

$$= \frac{(K+1)(K+1)}{(K+1)(K+2)}$$

$$= \frac{(K+1)}{(K+2)}$$

continued on next page

Question #31 (Score Point 4 Response) ***continued***

So the statement is true for $n = K + 1$.

Therefore, since both the initial and inductive steps have been completed, by induction, the statement is true for all natural numbers n.

Question #31 (Score Point 3 Response)

Assume the statement is true for $n = K$, then show that it is true for $n = K + 1$.

For $n = K$:

$$\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{K(K+1)} = \frac{K}{K+1}$$

Show true for $n = K + 1$, i.e.

$$\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{(K+1)(K+2)} = \frac{(K+1)}{(K+2)}$$

$$\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)} = \frac{K}{K+1} + \frac{1}{(K+1)(K+2)}$$

$$= \frac{K(K+2)+1}{(K+1)(K+2)}$$

$$= \frac{K^2 + 2K + 1}{(K+1)(K+2)}$$

$$= \frac{(K+1)(K+1)}{(K+1)(K+2)}$$

$$= \frac{(K+1)}{(K+2)}$$

Therefore, by induction, the statement is true for all n.

Question #31 (Score Point 2 Response)

Show the statement is true for $n = 1$:

left-hand side: $\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{n(n+1)} = \frac{1}{1(1+1)} = \frac{1}{2}$

right-hand side: $\frac{n}{n+1} = \frac{1}{1+1} = \frac{1}{2}$

$\therefore$ true for $n = 1$.

Assume the statement is true for $n = k$:

$$\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{k(k+1)} = \frac{k}{k+1}$$

Show true for $n = k + 1$:

$$\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \quad + \frac{1}{(k+1)(k+2)} = \frac{(k+1)}{(k+2)}$$

$\therefore$ by induction, the proof is complete.

Question #31 (Score Point 1 Response)

$$\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

For $n = 1$: $\frac{1}{2} = \frac{1}{2}$

For $n = 2$: $\frac{2}{3} = \frac{2}{3}$

For $n = k$: $\frac{k}{k+1} = \frac{k}{k+1}$

Therefore, by induction, the statement is true.

Scoring Information for CSET: Mathematics Subtest I

Responses to the multiple-choice questions are scored electronically. Scores are based on the number of questions answered correctly. There is no penalty for guessing.

There are four constructed-response questions in Subtest I of CSET: Mathematics. Each of these constructed-response questions is designed so that a response can be completed within a short amount of time—approximately 10–15 minutes. Responses to constructed-response questions are scored by qualified California educators using focused holistic scoring. Scorers will judge the overall effectiveness of your responses while focusing on the performance characteristics that have been identified as important for this subtest (see below). Each response will be assigned a score based on an approved scoring scale (see page 41).

Your performance on the subtest will be evaluated against a standard determined by the Commission on Teacher Credentialing based on professional judgments and recommendations of California educators.

Performance Characteristics for CSET: Mathematics Subtest I

The following performance characteristics will guide the scoring of responses to the constructed-response questions on CSET: Mathematics Subtest I.

PURPOSE	The extent to which the response addresses the constructed-response assignment's charge in relation to relevant CSET subject matter requirements.
SUBJECT MATTER KNOWLEDGE	The application of accurate subject matter knowledge as described in the relevant CSET subject matter requirements.
SUPPORT	The appropriateness and quality of the supporting evidence in relation to relevant CSET subject matter requirements.
DEPTH AND BREADTH OF UNDERSTANDING	The degree to which the response demonstrates understanding of the relevant CSET subject matter requirements.

Scoring Scale for CSET: Mathematics Subtest I

Scores will be assigned to each response to the constructed-response questions on CSET: Mathematics Subtest I according to the following scoring scale.

SCORE POINT	SCORE POINT DESCRIPTION
4	**The "4" response reflects a thorough command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.** • The purpose of the assignment is fully achieved. • There is a substantial and accurate application of relevant subject matter knowledge. • The supporting evidence is sound; there are high-quality, relevant examples. • The response reflects a comprehensive understanding of the assignment.
3	**The "3" response reflects a general command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.** • The purpose of the assignment is largely achieved. • There is a largely accurate application of relevant subject matter knowledge. • The supporting evidence is adequate; there are some acceptable, relevant examples. • The response reflects an adequate understanding of the assignment.
2	**The "2" response reflects a limited command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.** • The purpose of the assignment is partially achieved. • There is limited accurate application of relevant subject matter knowledge. • The supporting evidence is limited; there are few relevant examples. • The response reflects a limited understanding of the assignment.
1	**The "1" response reflects little or no command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.** • The purpose of the assignment is not achieved. • There is little or no accurate application of relevant subject matter knowledge. • The supporting evidence is weak; there are no or few relevant examples. • The response reflects little or no understanding of the assignment.
U	**The "U" (Unscorable) is assigned to a response that is unrelated to the assignment, illegible, primarily in a language other than English, or does not contain a sufficient amount of original work to score.**
B	**The "B" (Blank) is assigned to a response that is blank.**